1. Consider the Landau free energy

$$L = \int d^d x \left[\frac{1}{2}(\nabla\phi)^2 + \frac{1}{2} r_0 \phi^2 + \frac{1}{n!} u_0 \phi^n \right]$$

   Use the Ginzburg criterion and dimensional analysis to find the upper critical dimension. (Assume n is even.)

2. Prove that $f(x, y) = x^3y^2 + x^2y^3$ is a generalized homogeneous function.

3. Consider a model equation of state that can be written as

$$h \sim aM(t + bM^2 2)^\theta; \quad 1 < \theta < 2; \quad a, b > 0.$$

   near the critical point. Find the exponents β, γ and δ and check that they obey the relationship $\gamma = \beta(\delta - 1)$.

4. (Requires plotting) Suppose you have measured the magnetization of nickel at different fields and temperatures. Try to fit the data to the scaling hypothesis by plotting the collapsed data on the computer. To begin with assume that you do not know the values of T_c, β, and δ and then compare your best fits to the known values $T_c = 627.2^o$C, $\beta = 0.368$, and $\delta = 4.22$.

Magnetic field	Temperature(K)			
($\times 10^4$ A m^{-1})	621.9	623.8	625.7	631.3
3.42	0.119	0.110	0.085	--
7.08	0.145	—	—	--
10.78	0.150	0.133	0.112	0.044
25.70	0.164	0.150	0.134	0.079
47.87	0.178	0.166	0.152	0.109
80.13	0.191	0.181	0.170	0.134
113.1	0.202	0.193	0.183	0.151
141.4	0.210	0.201	0.192	0.163

Figure 1: Magnetization data for nickel in units of μ_B per atom [Weiss and Forrer (1926)]. You can download the data following this link!